Seat No.: ________ Enrolment No.___________

GUJARAT TECHNOLOGICAL UNIVERSITY

BE - SEMESTER– III (OLD) EXAMINATION – SUMMER 2022

Subject Code:130001 **Date:08-07-2022**

Subject Name:Mathematics-III

Time:02:30 PM TO 05:30 PM **Total Marks:70**

Instructions:

1. **Attempt all questions.**
2. **Make suitable assumptions wherever necessary.**
3. **Figures to the right indicate full marks.**
4. **Simple and non-programmable scientific calculators are allowed.**

Q.1 **(a)** (i) Solve $y' + y\sin x = e^{\cos x}$ **03**

(ii) Solve $(x^2 - x - y^2)dx - 2xy\,dy = 0$ **04**

(b) Find the power series solution of the equation $\frac{d^2y}{dx^2} + xy = 0$. **07**

Q.2 **(a)** (i) Solve $y'' + 9y = \cos 4x$ **03**

(ii) Using the method of variation of parameter, solve $y'' - 3y' + 2y = e^x$ **04**

(b) Using the method of undetermined coefficient, solve $y'' - 2y' = e^x \sin x$ **07**

OR

(b) Solve the equation by series method $4xy'' + 2y' + y = 0$ **07**

Q.3 **(a)**

(1) Show that $\int_0^1 x^2(1-x)^3\,dx = \frac{1}{60}$. **03**

(2) Prove that $\frac{d}{dx}\left(x^n J_n(x)\right) = x^n J_{n-1(x)}$ **04`**

(b) Find the Fourier series of $f(x) = x\sin x$ in the interval $(-\pi, \pi)$. Hence, deduce that $\frac{\pi - 1}{4} = \frac{1}{1\cdot 3} - \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} - \dots$ **07**

OR

Q.3 **(a)** Find the Find the Fourier series of $f(x) = x + x^2$ in the interval $(-\pi, \pi)$. Hence, deduce that $\frac{\pi^2}{6} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots$ **07**

(b) Find the Half range Fourier cosine series of $f(x) = x,\ 0 \le x \le \pi.\ \ f(x + 2\pi) = f(x)$ **07**

Q.4 **(a)** (1) Find the Laplace transform of the function $f(t) = t\sin 2t$. **03**

(2) Find the inverse Laplace transform of the function $F(s) = \frac{4s + 12}{s^2 + 8s + 16}$. **04**

(b) Solve the differential equation using Laplace Transformation method $\frac{d^2y}{dt^2} + y = t$, Given that $y(0) = 1,\ y'(0) = 0$, $t > 0$. **07**

OR

Q.4 **(a)** (1) Find the Laplace transform of the function $f(t) = e^{-2t}(t^2 + \sin 4t)$ **03**

(2) Find the inverse Laplace transform of the function $F(s) = \tan^{-1}\frac{2}{s}$ **04**

(b) Define Convolution theorem for Laplace transform. Using Convolution theorem to find Laplace inverse of the function $F(s) = \frac{1}{s^2(s+1)^2}$ **07**

Q.5 **(a)** (1) Form the partial differential equation of $z = axy + b$. **03**

(2) Solve $p - q = \ln(x + y)$ **04**

(b) Solve by the method of separation of variables $\frac{\partial^2 u}{\partial x^2} = \frac{1}{2}\frac{\partial u}{\partial t}$ over $0 < x < 3,\ t > 0.$ for the boundary conditions $u(0,t) = u(3,t) == 0$ and initial conditions $u(x,0) = 5\sin 4\pi x$ **07**

OR

Q.5 **(a)** (1) Solve the partial differential equation $y = e^q$ **03**

(2) Solve yp + xq + pq = 0 **04**

(b) Using Charpit's method to solve the $z^2 = pqxy$ **07**

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